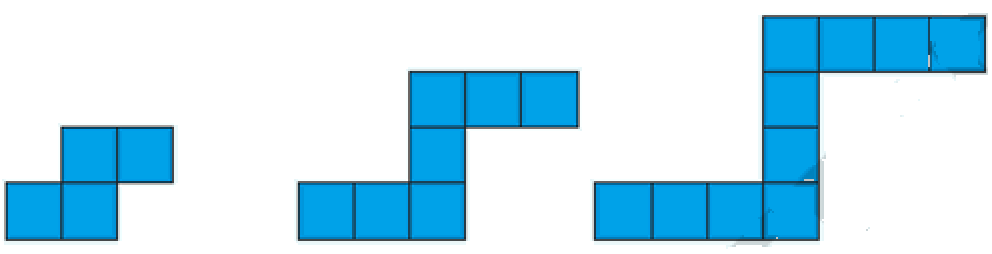
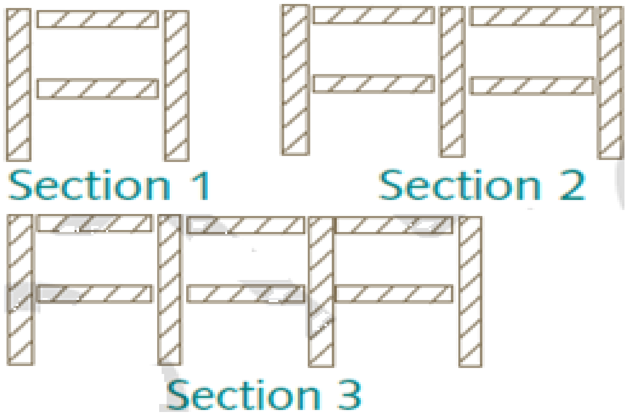
DEPARTMENT OF APPLIED SCIENCES (MATHEMATICS)

Assignment No. 2	
Title of Course: Discrete Structures	
Course Code: AML4209	
Topic Name: Recurrence Relations, P & C , Generating functions and Pigeonhole Principle	

1. Many horror stories involve zombie outbreaks. Suppose a zombie outbreak were to occur on Planet Earth. 13 zombies are unleashed and when they bite humans, the human immediately turn into zombies. The zombie population triples every hour. Write a recurrence relation for Z_n, the zombie population n hour after the outbreak and hence solve it.
2. For a photo, the staff at a company have been arranged such that there are 10 people in the front row and each row has 7 more people than in the row in front of it. Write a recursive formula, a_n, that completely defines the number of people in the n row. Write an explicit formula, a_n, for the number of people in the n^{th} row.
3. Solve the recurrence relation: $a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r$.
4. Solve the following recurrence relation $t_n = t_{n-1} + n, t_1 = 4$. and indicate if it is a linear homogeneous relation or not. If yes, give its degree and if not justify your answer.
5. What is the minimum number of students required in a class to be sure that at least 6 will receive the same grade if there are five possible grades A, B,C, D and F?
6. It is given that White tiger population in Orissa (India) is 30 at time $n = 0$ and 32 at time $n = 1$. Also the increase from time $n - 1$ to n is twice the increase from time $n - 2$ to time $n - 1$. Write the recurrence relation for growth rate of tiger and then solve it.
7. The number of bacteria in a colony doubles every hour. If a colony begins with 5 bacteria, how many bacteria will be there after n hours? Find a recurrence relation to represent the same and hence solve it.
8. Obtain and solve a recurrence relation for Fibonacci sequence.
9. The snake shapes below are made using blocks, each with a side length of 1 unit. The perimeter of each snake shape can be found by counting the sides of the blocks around the outside of the shape. Construct a recurrence relation for perimeter of this problem and hence solve the recurrence relation.



10. By using pigeonhole principle, show that if any five numbers from 1 to 8 are chosen, then two of them will add up to 9.
11. A single section of fencing is made from four logs. Two sections use seven logs. Examples of one, two and three-sections fences are shown below.



Construct a recurrence relation for the above problem and hence solve it.

12. There are 15 points in a plane, out of which 6 are collinear
    (i) How many straight lines can be formed by joining them?
    (ii) How many triangles can be formed by joining them?
13. State Tower of Hanoi puzzle and hence frame and solve recurrence relation for the problem.
14. Obtain partial fraction decompositions and identify the sequence having the expression as a generating function $\frac{6-29z}{1-11z+30z^2}$.
15. If U(n) = 2 . 3n, n ≥ 0, then G(U, z) is an infinite geometric series 2+2.3z+2.(3z)2+……. With

    a = z, r=3z.

16. Find the generating function and sequence of the recurrence relation $a_n+3a_{n-1} = 0$ with $a_0 = 7$.

    Write the Generating function of the sequence $S_n = 4.3^n + 5(-1)^n + 9$.

17. Find the least number of cables required to connect 100 computers to 20 printers to guarantee that 20 computers can directly access 20 different printers. Justify your answer.
18. A student must take 5 classes from 3 areas of study. Various classes are offered in each discipline but the student cannot take more than two classes in any given area. Using the pigeonhole principle, show that the student will take at least one class in each area.
19. Show that if 9 colors are used to paint 1000 houses, atleast 112 houses will be of the same color.
20. Show that if any 4 numbers 1 to 6 are chosen, then two of them will add to 7.
21. Find generating function of sequence 1, 2/3, 3/9, 4/27, …….
22. Find the value of $1^2+2^2+3^2+\ldots\ldots\ldots+n^2$ using generating function.
23. If S(k) – 4S(k-1)+3S(k-2) = k^2, then find the generating function of S, G(S,z) ?
24. Solve S(k) – 8S(k-1)+21S(k-2)-18S(k-3)=0 for n>=3 using generating function upto partial fractions.
25. Among a set of 5 black balls and 3 red balls, how many selections of 5 balls can be made such that at least 3 of them are black balls.
26. How many 4 digit numbers that are divisible by 10 can be formed from the numbers 3, 5, 7, 8, 9, 0 such that no number repeats?

27. In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?
28. A box contains 4 different black balls, 3 different red balls and 5 different blue balls. In how many ways can the balls be selected if every selection must have at least 1 black ball and one red ball?
29. In how many ways can 10 engineers and 4 doctors be seated at a round table if no two doctors sit together?
30. A company has 11 software engineers and 7 civil engineers. In how many ways can they be seated in a row so that all the civil engineers do not sit together?
31. Solve the recurrence relation $a_r - 7a_{r-1} + 10a_{r-2} = 0$ by the method of generating functions with the initial conditions$a_0 = 1, a_1 = 2$.
32. Find the generating function for the following sequences. In each case try to simplify the answer.
    a) 1,1,1,1,1,1,0,0,0,0….
    b) 1, 1, 1, 1, 1…..
    c) 1,3,3,1,0,0,0,0,…..
    d) $C_0^{2005}, C_1^{2005}, C_2^{2005}, \dots \dots .., C_{2005}^{2005}, 0,0,0,0, \dots \dots$