CHITKARA UNIVERSITY

DEPARTMENT OF APPLIED SCIENCES (MATHEMATICS)

Assignment No. 1
Title of Course: Discrete Structures
Course Code: AML4209
Topic Name: Sets, Relations and Functions

1. Which of the following relations are Symmetric?
   (a) 'is a sister' on the set of all members of a family.
   (b) 'is a multiple of' on the set of N.
   (c) 'is a devisor of' on the set of Integers.
   (d) 'is perpendicular to' on the set of lines on a plane.
2. In a survey of 500 television watchers 285 watch Football, 195 watch Hockey, 115 watch Basketball, 45 watch Football and Basketball, 70 watch Football and Hockey, 50 watch Hockey and Basketball, 50 do not watch any of the three games.
   (a) How many watch all the three games.
   (b) How many watch exactly one of the three games.
   (c) How many watch Football but not Basketball and Hockey.
3. In a town of 10,000 families, it was found that 40% families buy newspaper A,20% buy newspaper B, 10% buy newspaper C,5% buy newspaper A and B,3% buy newspaper B and C,4% buy newspaper C and A and 2% buy all the three newspapers. Find the number of families which buy newspaper
   (a) A only
   (b) B only
   (c) None of A,B and C
4. 100 of 120 students at a college take at least one of the languages Hindi, English and French.65 study Hindi, 45 study English, 42 study French, 20 study English and Hindi,25 study Hindi and French and 15 study English and French. Find the number of students studying all subjects. Also find number of students studying exactly one subject.
5. Give an example of a relation which is reflexive but neither symmetric nor transitive.
6. Give an example of a relation which is symmetric but neither reflexive nor transitive.
7. If R and S be the following relations on A = { 1,2,3 }
   R^{-1} = {(1,1),(1,2),(2,3),(3,1),(3,3)}, S = {(1,2),(1,3),(2,1),(3,3)}
   Find (a) R ∩ S, R ∪ S, (b) RoS (c) S2 = SoS.
8. Let A= {1, 2, 3} and B={a,b,c,d}. In each case, state whether the given functions (if defined) is Injective, surjective, bijective.
   (a) f={(1,a),(2,d),(3,d)}
   (b) g={(1,a),(1,b),(2,d),(3,c)}
   (c) h={(1,a),(2,b)}
9. In a class of 40 students, 12 enrolled for both English and German. 22 enrolled for German. If the Students of the class enrolled for at least one of the two subjects, then how many students enrolled for only English and not German?
10. In a survey it was found that, the number of people that like only Pepsi, only Coke, both Coke and Pepsi and neither of them are 2n, 3n, 69/n, 69/3n respectively. Find n?

11. Find the domain and range of f(x) = $\frac{x}{x-2}$

12. Find the domain and range of f(x) = $\frac{x^2-2}{x^2+2}$

13. If R is a relation defined as aRb, iff | a-b|>0, a ,b> 0 then the relation is
(a) Reflexive (b) symmetric (c) transitive (d) symmetric and transitive

14. If R is the relation in NXN defined by (a,b)R(c,d) iff a+d=b+c, show that R is an equivalence Relation.

15. Among integer 1 to 300, how many of them are divisible by 3, nor by 5, nor by 7? How many of Them are divisible by 3 but not by 5 nor by 7?

16. Let A={1,2,3,4,5,6} Define a relation R on A, R ={(x,y):x+y is a divisor of 24}
(i) Write elements of relation. (ii) Find the relational matrix M_R. (iii) Discuss its properties.

17. The loudness of sound measured in decibels (dB) varies inversely as the square of the distance Between the listener and the source of the sound. If the loudness of sound is 17.92 dB at a distance of 10 ft from a stereo speaker, what is the decibel level 20 ft from the speaker?

18. A function f is defined on the set of integers as follows

$$f(x) = \begin{cases} 1+x & 1 \le x \le 2 \\ 2x-1 & 2 \le x \le 4 \\ 3x-10 & 4 < x \le 6 \end{cases}$$

(i) Find the domain and range of the function.
(ii) Find the value of $f(4)$.
(iii)State whether f is one-one or many one function.

19. Find the domain and range of the following functions:
(i) f(x)= $\sqrt{16-x^2}$
(ii) f(x)= $\frac{1}{(x-1)(x-2)}$

20. What is the sum of all integers from 1 to 100 that are multiples of 2 or 3?

21. A veterinarian surveys 26 of his patrons. He discovers that 14 have dogs, 10 have cats, and 5 have fish. Four have dogs and cats, 3 have dogs and fish, and one has a cat and fish. If no one has all three kinds of pets, how many patrons have none of these pets?

22. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?

23. Given three sets P, Q and R such that:
P = {x: x is a natural number between 10 and 16},
Q = {y: y is a even number between 8 and 20} and
R = {7, 9, 11, 14, 18, 20}
(a) Find the difference of two sets P and Q
(b) Find Q – R
(c) Find R - P

(d) Find Q – P

24. Prove that (mod m) is an equivalence relation iff $a \equiv b(\text{mod } m)$ where $a, b \in I$.

25. Let A={1,2,3,4,5,6,7,8,9,10} .Find all min terms generated by B={4,5,6},C={7,8,9} and D={1,2,3}.

26. Salad is made with combination of one or more eatables, how many different salads can be prepared from onion, tomato, carrot, cabbage and cucumber.

27. If A={ +,-} and B={ 00,01,10,11}

(1) Find A X B

(2) How many elements of A^4 and $(A \times B)^3$ have?

28. Consider A={1,2,3,4,5,6,7,8,9}. Let B_1={5,6,7} , B_2={2,4,5,9} and B_3={3,4,5,6,8,9}. Find the minsets generated by B_1, B_2, B_3.

(a)Do these minsets form a partition of A.

(b) How many different subsets of A can you create using B_1, B_2, B_3 with standard set operations.

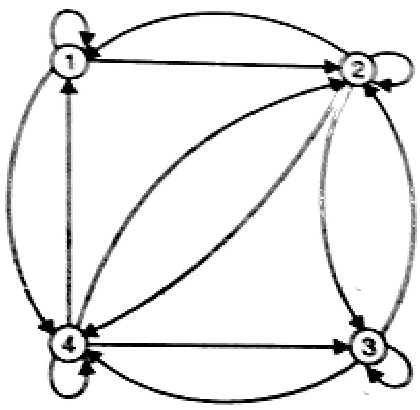
29. Give example of relation which is:

(a) neither reflexive nor irreflexive

(b) both symmetric and anti-symmetric

(c) each reflexive, symmetric and transitive.

30. Let R be a relation on A whose corresponding directed graph is given below:



Determine the matrix on R.

31. Let $A = \{1, 2, 3\}$ and R and S be relations on A. Suppose the matrices of R and S are

$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Find

(a) M_{RoS} (b) $M_{R\cup S}$ (c) $M_{R\cap S}$ (d) $M_{R^{-1}}$

32. Given $A = \{1, 2, 3, 4\}$ Consider the following relation in A

$R = \{(1,1), (2,2), (2,3), (3,2), (4,2), (4,4)\}$

(a) Draw its directed graph.

(b) Is R (i) Reflexive (ii) Symmetric (iii) Transitive (iv) anti-symmetric?

(c) Find $R^2 = RoR$.

33. Show that a function $f(x) = \frac{x+1}{x-3}$ is bijective. If yes, then find its inverse.

34. Let f, g and h be functions from N to N, where N is the set of natural numbers so that

$f(n) = n + 1, g(n) = 2n, h(n) = \begin{cases} 0, & when\ n\ is\ even \\ 1, & when\ n\ is\ odd \end{cases}$

Determine fof, fog, gof, goh, hog, (fog)oh.

35. Is $f(x) = \dfrac{x-1}{x+1}$ is invertible in its domain? If so find f^{-1} .Also verify $fof^{-1}(x) = x$.

36. If A has 4 elements and B has 3 elements. Then find

a) Number of relation from A to B

b) Number of functions from A to B

c) Number of injective functions from A to B

37. If U={1,2,3,4,5,6}, A={1,2,3,4},B={3,4,5,6}.Find the bit string for the set A and B and use bit string to find A^c and union and intersection of sets A and B.

38. How many integers between 1 and 60 that are not divisible by 2 nor by 3 and nor by 5.Also

Determine the number of integers divisible by 5, not by 2, not by 3.

39. How many elements in (A x B) and (B x A) are common if n elements are common to A and B?

40. If R be a relation in the set of integers Z defined by R = {(x-y): x, y Є Z, (x-y) is multiple of 3}. Show that it is an equivalence relation.

41. If f: N→N given by f(n) = 2n if n is even and f(n) = n if n is odd. Check whether f is one-one?

42. Determine whether the function f : R→R given below are 1-1 and onto R

  i. f(x)=x+1
  ii. $f(x)=x^2$
  iii. f(x)=|x|+x for all x Є R
  iv. $f(x)=x^3$

43. The function f : R→R given by f(x)=2x+1 for $0 \leq x < 2$
=x-2 for $2 \leq x < 5$

Find domain and range of f. Whether the function is 1-1 or many one?

44. If A is a collection of sets. Check whether the relation of "Subset" on A is Partial Order Relation or not?

45. Let A = {1, 2, 3, 4, 5, 6}. Define a relation R on A and R = {(x, y): x +y is a divisor of 24}.

  i. Find the relational Matrix M of R.
  ii. Compute M^2 and use M and M^2 whether or not R is transitive.

46. Let R be the relation from S = {1, 2, 3, 4} to T = {a, b, c} with Boolean Matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

  i. Check RoR^{-1} is a symmetric relation on S.
  ii. Are the relation $R^{-1}oR$ and RoR^{-1} equivalence relation.

47. A computer 350 applications from computer graduates for a job planning a new line of web Servers. Suppose 220 of these people majored in computer science, 147 majored in business and 51 majored both in computer science and business. How many of these majored neither in computer science or business.

48. Use Ceiling function, How many bytes are required to encode n bits of data for the following values of n:

  a) 7
  b) 17
  c) 1001
  d) 28800

49. If R is the relation on N x N defined by (a, b) R (c, d) if and only if ad = bc, show that R is an equivalence relation.

50. Let S = {red, blue, green, yellow}. Determine whether or not each of the following is a partition of S.

  i. P_1={{red}, {blue, green}}
  ii. P_2={{red, blue, green, yellow}}
  iii. P_3={{S}}

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